SIMPLE EQUIVALENT CIRCUIT MODELING OF SMALL APERTURES IN TRANSMISSION LINE MATRIX (TLM) METHOD

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Abstract

A technique to incorporate a small aperture model into the Transmission Line Matrix (TLM) code based on Bethe's small hole coupling theory [1] is presented. Electrically small apertures are modeled as electric and magnetic dipoles. A link between these equivalent dipoles and a simple equivalent circuit is found and implemented in the TLM mesh.

The method has been validated by calculating the first resonant frequency of two cavities coupled by a narrow aperture, and by comparing the results with those obtained by TLM analysis with increasingly denser meshes. The obtained results confirm the validity of the approach and the high potentiality in terms of accuracy and savings in memory and CPU time requirements.

Introduction

Small apertures in a conductive wall are particularly difficult to model in space discrete methods since a high mesh resolution is required in order to achieve a good field description inside the aperture. Such small apertures, however, are very common in EMC/EMI problems, antenna arrays, and waveguide couplers.

When the dimensions of the aperture are much smaller than the other dimensions of the structure under test, discretization of the aperture leads to a mesh with cells that vary considerably in size throughout the structure. As a consequence, the required time step become extremely small. If the aperture were not present, a cell size much larger than the aperture dimension would be sufficient.

Unless the spatial cell size is reduced down to that required to resolve the aperture, alternative methods must to be used to characterize the aperture. Several solutions have been proposed in the past for both Finite-Difference Time-Domain (FDTD) and Transmission Line Matrix (TLM) method [2-6].

In this paper a novel and effective way to introduce a small aperture model into the TLM standard algorithm is described. The presence of a small aperture is modeled by introducing an equivalent circuit into the TLM network. The circuit parameters are derived from the electric and magnetic polarizabilities associated with the geometrical dimensions of the aperture. In this way a single cell can be placed on the aperture while maintaining a high degree of accuracy.

(Theoretical Background)

According to Bethe's theory [1], a small aperture in a conducting wall can be approximated by the combination of an electric dipole normal to the aperture and proportional to the normal component of the exciting electric field, and a magnetic dipole in the plane of the aperture that is proportional to the exciting tangential magnetic field.

The proportionality constants depend on the aperture size and shape. These constants are referred to as the electric and magnetic polarizabilities of the aperture. This situation is represented in Fig. 1. The equivalent dipoles radiate in the presence of the closed conducting wall to give the fields transmitted through the aperture. The fields on the input side of the conducting wall are also affected by the presence of the aperture, and this effect is accounted for by the equivalent dipoles on the incident side of the conductor, with opposite sign. Using the image theory the electric wall can be removed, doubling the dipole amplitudes.

The electric and magnetic dipoles $\mathbf{P}_e$ and $\mathbf{P}_m$ are:

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$$P_e = -\varepsilon_0 \alpha_e (n \cdot E)\, n$$
$$P_m = -\alpha_m H_t \quad (1)$$

where $n \cdot E$ is the normal electric field and $\mathbf{H}_t$ is the tangential magnetic field evaluated at the center of the aperture. The electric and magnetic polarizabilities α_e, α_m, are constants that depend on the size and shape of the aperture, and have been derived for a variety of simple shapes [7], [8], [9]. The polarizabilities for circular and rectangular apertures, which are among the most commonly used shapes, are given in Table 1

Table 1: Electric and Magnetic Polarizabilities

Aperture Shape	α_e	α_m
Round Hole	$\frac{2}{3} r_0^3$	$\frac{4}{3} r_0^3$
Rectangular Slot (H across slot)	$\frac{\pi}{16} l d^2$	$\frac{\pi}{16} l d^2$

where r_0 is the radius of the circular hole, l and d are respectively the length and width of the rectangular slot. The electric and magnetic polarization currents, $\mathbf{P}_e$ and $\mathbf{P}_m$, can be related to electric and magnetic current sources, **J** and **M** respectively. We can write:

$$\nabla \times E = -j\omega\mu_0 H - j\omega\mu_0 P_m$$
$$\nabla \times H = j\omega\varepsilon_0 E + j\omega P_e \quad (2)$$

2D-TLM: Modeling of a Slot

Consider a narrow slot aperture in the transverse wall of a waveguide as shown in Fig. 2.

The analysis of this problem can be performed using the two-dimensional TLM shunt node, which models the field components E_y, H_x, H_z. Since H_x is the only magnetic field tangential to the aperture and there are no electric fields normal to the aperture, the electric polarization current $\mathbf{P}_e$ is equal to zero, and the magnetic polarization current $\mathbf{P}_m$ has only a x-component P_{xm}. Therefore, according to (1), P_{xm} is given by:

$$P_{xm} = -\alpha_m H_x \delta(x - x_0)\, \delta(y - y_0)\, \delta(z - z_0) \quad (3)$$

From Maxwell's equations we have:

$$\frac{\partial E_y}{\partial z} = j\omega\mu_0 H_x + j\omega\mu_0 P_{xm} \quad (4)$$

Equation (4) suggests that the aperture can be modeled with only one TLM cell, provided that a magnetic polarization current P_{xm} is introduced. This is equivalent to altering locally the value of the magnetic permeability μ_r by a factor proportional to the magnetic polarizability α_m of the aperture.

In terms of the TLM algorithm this amounts to adding a short-circuited stub in parallel to the link line passing through the aperture. In order to preserve the synchronicity of the TLM scheme the length of the stub is set to $\Delta l/2$ and the value of μ_r is determined by selecting the characteristic impedance Z_x of the stub. Given the desired value μ_r, the extra inductance which must be provided by the stub is $L_s = (\mu_r - 1)\mu_0 \Delta l/2$.

The quantity $(\mu_r - 1)$ is the magnetic susceptibility χ_m associated with the magnetic polarization P_{xm}. Considering the image effect of the closed wall, which doubles the value of P_{xm}, we have:

$$\chi_m = 2\frac{\alpha_m}{\Delta l^3} \quad ; \quad L_s = \mu_0 \frac{\alpha_m}{\Delta l^2} \quad (5)$$

The value of Z_x is immediately derived from (5), considering the input impedance formula for a short circuited stub $jZ_x \tan\left(\beta\frac{\Delta l}{2}\right) = j\omega L_s$.

$$Z_x = 2\frac{\alpha_m}{\Delta l^3} Z_0 \quad (6)$$

This extra term can be introduced in two ways:

•Adding a stub, by introducing a local scattering process at each propagation time-step

•Directly modeling the inductance L_s, by means of a discretization of the operator $j\omega L_s$.

They both can be implemented in the TLM algorithm very easily. The latter solution has been chosen [10]. The two alternatives are reported in Fig. 3.

Fig. 3 (a) depicts a short-circuited stub parallel to the TLM link lines, while Fig. 3 (b) shows the introduction of the equivalent inductance on the TLM link lines. The two link lines at ports 1 and 2 are cascaded with the discretized scattering matrix of the shunt inductance.

Results

In order to test the proposed methods we have calculated the first resonant frequency of the cavity shown in Fig. 4. The cavity is divided in two sub-chambers coupled by a narrow aperture. We have first analyzed the cavity using the standard TLM algorithm with four increasingly finer discretizations. Then we have modeled the aperture with its equivalent inductance using only the coarsest of the four meshes, that is with the aperture width equal to one single cell.

The frequency domain results are shown in Fig. 5. In order to evaluate the effectiveness of the model we have considered, as a reference value, the resonant frequency extrapolated from the results for $\Delta l \to 0$ by using Richardson extrapolation [11]. This reference value has been used to evaluate the relative error. All the results are reported in Table 2

Table 2: Narrow Slot in a Cavity: Resonant Frequencies (GHz) and Comparisons

Δl (mm)	1	1/3	1/5	1/11	**Δl ->0**
TLM	47.57	50.22	50.63	50.94	**51.20**
Err. (%)	**7.0%**	1.9%	1.1%	0.5%	
Bethe	50.75				
Err. (%)	**0.9%**				

The accuracy of the simulation is highly improved by the introduction of the equivalent dipole-based model. Moreover, slots of arbitrarily width and shape can be promptly simulated without the need of changing the discretization of the rest of the structure, thus adding a large flexibility and efficiency to the TLM method.

Conclusions

A novel systematic procedure to incorporate a small aperture model into a TLM mesh has been proposed. The method is based on the derivation of an equivalent circuit model for the aperture obtained from Bethe's small hole coupling theory. The procedure requires negligible additional operations and leads to a saving in computational time and memory of three orders of magnitude as compared to direct discretization of the aperture.

The technique can be extended to handle apertures of various shapes such as diamonds, circles, and rounded end slots.

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Figures

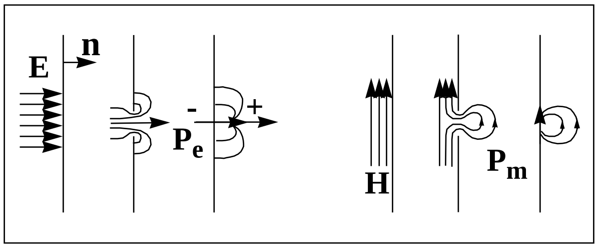

Fig. 1 Aperture in a conducting wall:
Equivalent electric and magnetic dipoles

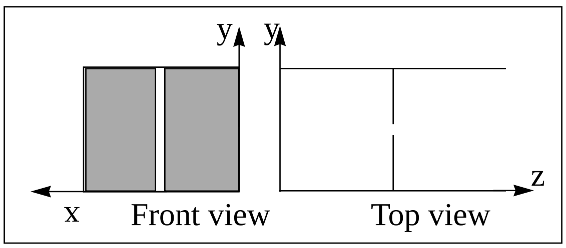

Fig. 2 Slot aperture

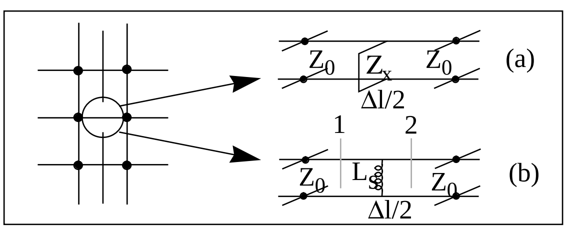

Fig. 3 Narrow Slot in a TLM Mesh

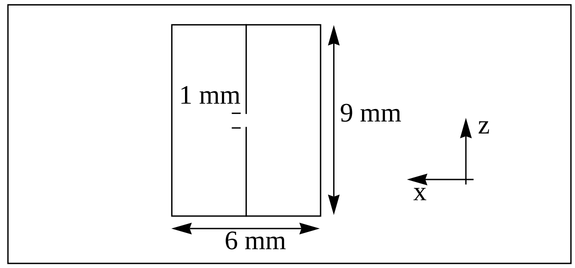

Fig. 4 Cavities Coupled by a Narrow Slot

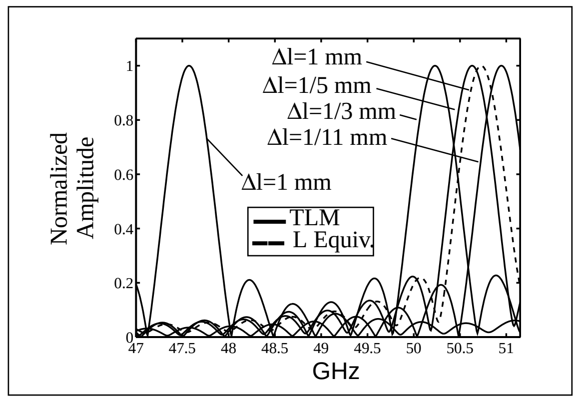

Fig. 5 Narrow Slot in a Cavity: Resonant Frequencies